Part 1, MULTIPLE CHOICE, 5 Points Each

1 Let

$$U = \{2, 4, 6, 8, 10, 12, 14, 16\}.$$
$$A = \{2, 4, 6, 8, 10\}$$
$$B = \{4, 8, 12, 16\}$$

Find $(A \cap B)'$.

(a) $\{12, 16\}$ (b) $\{2, 6, 10\}$ (c) $\{2, 6, 10, 12, 14, 16\}$ (d) $\{14\}$ (e) $\{4, 8\}$

2 If R and S are finite subsets of a universal set U, such that

$$n(R') = 20, \quad n(S) = 15, \quad n(S' \cap R') = 5 \quad \text{and} \quad n(U) = 35,$$

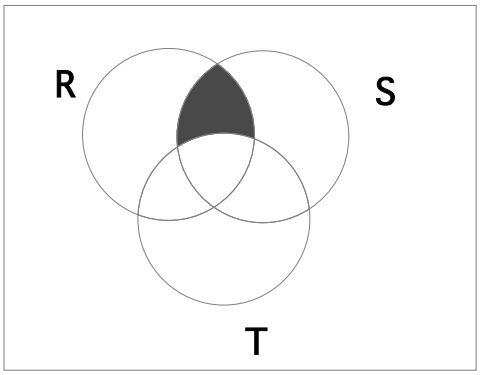
how many elements in $S \cap R$.

(a) 5 (b) 10 (c) 25 (d) 0 (e) 15

3 A survey of 68 people showed that 50 liked Frosted Flakes, 49 liked Cheerios and 46 liked Lucky Charms. 36 liked both Frosted Flakes and Lucky Charms, 33 liked Cheerios and lucky charms and 39 liked Cheerios and Frosted Flakes. Twenty Seven liked all three types of cereal. How many didn't like any of the three cereals.

(*a*) 4 (*b*) 0 (*c*) 2 (*d*) 10 (*e*) 7

4 Identify the shaded region in the Venn diagram below.



(*a*) $R \cap S \cap T$ (*b*) $R \cap S \cap T'$ (*c*) $R \cap S$ (*d*) $(R \cap S) \cup T'$ (*e*) $(R \cap T') \cup S$

5 Five square tiles of the same size but of different colors (all 5 colors are different) are arranged side by side in a horizontal line. How many different patterns are possible?

(a) 2^5 (b) 5 (c) 5^2 (d) 120 (e) 100

6 A chess club consisting of 20 members must choose a president, a secretary and a treasurer. If every club member is eligible for every position and positions cannot be shared, in how many ways can the above three officers be chosen?

(a) $P(20,3)$ (b) 20^3 (c) $C(20,18)$ (d) 3^{20} (e) $20+19+18$

7 An urn contains 7 numbered balls, 3 red and 4 green. A sample of 3 balls is selected from the urn. How many such samples with 2 red balls an 1 green ball are possible?

(*a*) $C(3,2)$ (*b*) $C(7,3)-4$ (*c*) $\frac{C(7,3)}{2!}$ (*d*) $C(3,2)\cdot C(4,1)$ (*e*) $C(4,1)$

8 A poker hand consists of a sample of 5 cards drawn from a deck of 52 cards. How many such hands have exactly three clubs?

Recall that a deck of cards has 13 clubs, 13 hearts, 13 diamonds and 13 spades.

(*a*) $C(13,3)+C(39,2)$ (*b*) $3\cdot C(39,2)$ (*c*) 13^3

(*d*) $C(13,3)\cdot C(49,2)$ (*e*) $C(13,3)\cdot C(39,2)$

9 Which of the following is equal to $C(1000, 850)$?

(a) $C(1000, 150)$ (b) $P(1000, 850)$ (c) $C(1001, 850) + C(1001, 851)$

(d) $P(1001, 850) + P(1001, 851)$ (e) $C(850, 1000)$

10 In a class of 75 math students we wish to assign each student to one of the projects, Project A (How to lie using statistics), Project B (How to use mathematics to choose the perfect Partner) or Project C (The mathematics of gambling). We will assign 25 students to work on Project A, 30 students to work on Project B and 20 students to work on Project C. In how many ways can the three teams of students be chosen?

(a) $P(75, 25) \cdot P(50, 30) \cdot P(20, 20)$ (b) $C(75, 25) + C(50, 30) + C(20, 20)$

(c) $C(75, 25) \cdot C(75, 30) \cdot C(75, 20)$ (d) $P(75, 25) \cdot P(75, 30) \cdot P(75, 20)$

(e) $C(75, 25) \cdot C(50, 30) \cdot C(20, 20)$

Part II, PARTIAL CREDIT,

Show all of your work for credit

11, How many 4 letter words can be made from the letters of the word

THURSDAY

(a) If letters CAN be repeated.

(b) If letters CANNOT be repeated.

(c) If the words must end in a vowel and letters CANNOT be repeated (NOTE: y is not a vowel).

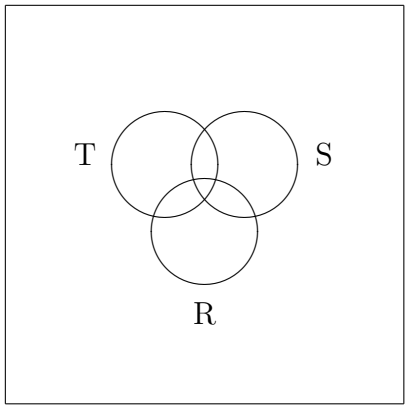
12, (a) Use De Morgan's laws to simplify

$$T' \cap (T' \cup S')'$$

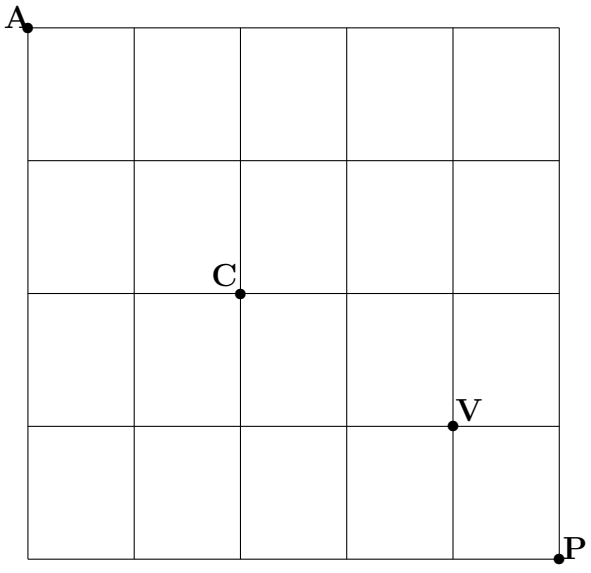
(b) Shade the region corresponding to

$$R \cap (S \cup T)'$$

in the diagram below.



13 A streetmap of Mathville is given below. You arrive at the Airport at A and wish to take a taxi to Paschal's house at P. The taxi driver, being an honest sort, will take a route from A to P with no backtracking, always travelling south or east.



(a) How many such routes are possible from A to P?

(b) If you insist on stopping off at the Combinatorium at C, how many routes can the taxi driver take from A to P?

(c) If wish to stop off at both the combinatorium at C and the Vennitarium at V, how many routes can your taxi driver take?

14, The following three yes/no questions were posed to a class of 68 students in a survey:

(i) Do You like Rap music?

(ii) Do You like Classical music?

(iii) Do You like Eighties music?

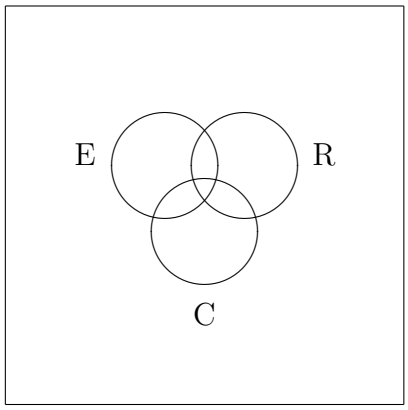
The results showed that 44 liked Rap music, 47 liked Classical music and 55 liked Eighties music. Nineteen students liked all three types of music, 7 liked Rap and Classical but not Eighties music, 19 liked Classical and Eighties music, but not Rap and 15 liked Rap and Eighties music, but not Classical.

(a) Present the Data given above on a Venn diagram, where R denotes the set of student's who like Rap, C denotes the set of student's who like Classical and E denotes the set of student's who like Eighties music.

(b) How many students didn't like any of the above music types?

(c) If a student was in the set $E \cap (R \cup C)'$, what answers did they give to questions (i), (ii) and (iii)?

(c) If a student answered in the following way: (i) yes, (ii) no, (iii) yes, place an X in the following Venn diagram to indicate which basic region corresponds to that student's preferences.



15, (a) Fill in the next line of Paschal's triangle below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

(b) If you flip a coin 6 times, the result is a sequence of H's and T's of length 6. How many such sequences have at least 3 heads? (Paschal's triangle above may help reduce the amount of work involved)